Linear Inequalities

Question1

Let S be the set of all real roots of the equation,
$3^x(3^x - 1) + 2 = 3^x - 1 | +3^x - 2 |$. Then S:
[Jan. 8, 2020 (II)]

Options:

A. contains exactly two elements.

B. is a singleton.

C. is an empty set.

D. contains at least four elements.

Answer: B

Solution:

Solution:
Let $3^x = y$
$\therefore\ y(y-1)+2 = |y-1|+|y-2|$
Case 1: when $y > 2$
$y^2 - y + 2 = y - 1 + y - 2$
$y^2 - 3y + 5 = 0$
$\because\ D < 0$[∴ Equation not satisfy.]
Case 2 : when $1 \le y \le 2$
$y^2 - y^2 + 2 = y - 1 - y + 2$
$y^2 - y + 1 = 0$
$\because\ D < 0$[∴ Equation not satisfy.]
Case 3: when $y \le 1$
$y^2 - y + 2 = -y + 1 - y + 2$
$y^2 + y - 1 = 0$
$\therefore y = \frac{-1+\sqrt{5}}{2}$
$= \frac{-1-\sqrt{5}}{2}$ [∴ Equation not Satisfy]
∵ Only one $-1 + \frac{\sqrt{5}}{2}$ satisfy equation

Question2

If $A = \{x \in R: |x| < 2\}$ and $B = \{x \in R: |x-2| \ge 3\}$; then :
[Jan. 9, 2020 (II)]

Options:

A. $A \cap B = (-2, -1)$

B. $B - A = R - (-2, 5)$

C. $A \cup B = R - (2, 5)$

D. $A - B = [-1, 2)$

Answer: B

Solution:

Solution:
$A = \{x : x \in (-2, 2)\}$
$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$
$A \cap B = \{x : x \in (-2, -1]\}$
$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$
$A - B = \{x : x \in (-1, 2)\}$
$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

Question3

Consider the two sets :
$A = \{ m \in R$: both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real $\}$ and $B = [-3, 5)$ Which of the following is not true?
[Sep. 03, 2020 (I)]

Options:

A. $A - B = (-\infty, -3) \cup (5, \infty)$

B. $A \cap B = \{-3\}$

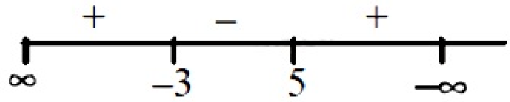
C. $B - A = (-3, 5)$

D. $A \cup B = R$

Answer: A

Solution:

Solution:
$A = \{ m \in R : x^2 - (m + 1)x + m + 4 = 0$ has real roots
$D \geq 0$
$\Rightarrow (m + 1)^2 - 4(m + 4) \geq 0$
$\Rightarrow m^2 - 2m - 15 \geq 0$



$A = \{(-\infty, -3] \cup [5, \infty)\}$
$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

Question4

The region represented by $\{z = x + iy \in C : |z| - Re(z) \leq 1\}$ is also given by the inequality:
[Sep. 06, 2020 (I)]

Options:

A. $y^2 \geq 2(x+1)$

B. $y^2 \leq 2\left(x+\frac{1}{2}\right)$

C. $y^2 \leq x+\frac{1}{2}$

D. $y^2 \geq x+1$

Answer: B

Solution:

Solution:
$\because |z| - \text{Re}(z) \leq 1 (\because z = x + iy)$
$\Rightarrow \sqrt{x^2+y^2} - x \leq 1 \Rightarrow \sqrt{x^2+y^2} \leq 1 + x$
$\Rightarrow x^2 + y^2 \leq 1 + x^2 + 2x$
$\Rightarrow y^2 \leq 1 + 2x \Rightarrow y^2 \leq 2\left(x+\frac{1}{2}\right)$

Question5

The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is : [Jan. 12, 2019 (II)]

Options:

A. 3

B. 8

C. 7

D. 6

Answer: C

Solution:

Solution:
Let the given quadratic expression
$(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, is positive for all $x \in R$
then
$1 + 2m > 0 \ldots$(i)
$D < 0$
$\Rightarrow 4(1+3m)^2 - 4(1+2m)4(1+m) < 0$
$\Rightarrow 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$
$\Rightarrow m^2 - 6m - 3 < 0$
$\Rightarrow m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$
From (i)
$\therefore m > -\frac{1}{2}$
$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$
Then, integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$
Hence, number of integral values of $m = 7$

Question6

The number of integral values of m for which the equation $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root is : [April 08, 2019 (II)]

Options:

A. 1

B. 2

C. infinitely many

D. 3

Answer: C

Solution:

Solution:
Given equation is
$(1+m^2)x^2-2(1+3m)x+(1+8m)=0$
$\because$ equation has no real solution
$\therefore D<0$
$\Rightarrow 4(1+3m)^2<4(1+m^2)(1+8m)$
$\Rightarrow 1+9m^2+6m<1+8m+m^2+8m^3$
$\Rightarrow 8m^3-8m^2+2m>0$
$\Rightarrow 2m(4m^2-4m+1)>0 \Rightarrow 2m(2m-1)^2>0$
$\Rightarrow m>0$ and $m\neq\frac{1}{2}$
$\left[\because \frac{1}{2}\right.$ is not an integer]
$\Rightarrow$ number of integral values of m are infinitely many.

Question7

All the pairs (x,y) that satisfy the inequality $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\le 1$ also satisfy the equation: [April 10, 2019 (I)]

Options:

A. $2|\sin x|=3\sin y$

B. $2\sin x=\sin y$

C. $\sin x=2\sin y$

D. $\sin x=|\sin y|$

Answer: D

Solution:

Solution:
Given inequality is,
$2\sqrt{\sin^2 x - 2\sin x + 5} \le 2^{2\sin^2 y}$
$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \le 2\sin^2 y$
$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \le 2\sin^2 y$
It is true if $\sin x = 1$ and $|\sin y| = 1$
Therefore, $\sin x = |\sin y|$

Question8

If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1, x \in R$, then the equation $f(x) = 0$ has :
[Online April 9, 2014]

Options:

A. no solution

B. one solution

C. two solutions

D. more than two solutions

Answer: B

Solution:

Solution:
$f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$
Put $f(x) = 0$
$\Rightarrow 0 = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$
$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$
$\Rightarrow 3^x + 4^x = 5^x \ldots$(i)
For $x = 1$
$3^1 + 4^1 > 5^1$
For $x = 3$
$3^3 + 4^3 = 91 < 5^3$
Only for $x = 2$, equation (i) Satisfy
So, only one solution $(x = 2)$

Question9

If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
[2002]

Options:

A. less than 1

B. equal to 1

C. greater than 1

D. anyreal no.

Answer: A

Solution:

$\because (a-b)^2+(b-c)^2+(c-a)^2>0$
$\Rightarrow 2(a^2+b^2+c^2-ab-bc-ca)>0$
$[\because a^2+b^2+c^2=1]$
$\Rightarrow 2>2(ab+bc+ca) \Rightarrow ab+bc+ca<1$
